This essay describes a tiny, counterintuitive change to the mathematics of the Schrődinger wave packet

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ABSTRACT

This 8 page essay proposes that a tiny change in the mathematics of a Schrődinger wave packet could vastly simplify how we perceive the world around us. Quantum experiments change their meaning. They no longer say the quantum world is weird. It is astonishing that such vast changes could result from such a tiny, almost insignificant change in math. We admit the change is counterintuitive, but that is an advantage, because it explains why Einstein and dozens of geniuses could not untangle quantum weirdness after a century of hard work. The mathematical change is to view a Schrődinger wave packet as part of a larger Elementary Wave traveling in the opposite direction. Equivalent changes would be made to QED and Quantum Field Theory. This essay is part of a scholarly article published in 2020: https://doi.org/10.24297/jap.v17i.8696

1 Introduction

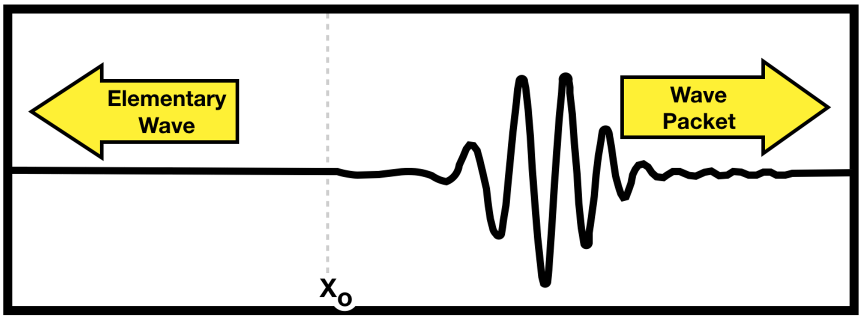

Figure 1: A one dimensional Elementary Wave (Æ) moves to the ← left, while a Schrődinger wave packet (Ψ) moves to the right →. They are two aspects of the same thing.

The Schrődinger wave packet in Fig. 1 consists of a wiggly line. Although the wave packet is moving to the right, in what direction is the line itself moving? One might assume that it is stationary. But consider a model in which the line itself is moving rapidly to the left. The Theory of Elementary Waves (TEW) proposes that the left side of the line in Fig. 1 is a one dimensional plane wave $\Psi(x) = e^{-i(kx-\omega t)}$ moving at light speed to the left. Under certain specialized circumstances a Schrődinger wave packet would pop out from inside this plane wave, and move to the right.[Boyd 2012 through 2020c; Little 1996, 2000 & 2009]

Such a plane wave that carries within it a trigger mechanism for the sudden emergence of a Schrődinger wave packet moving in the opposite direction, is called an "Elementary Wave" which we denote by the letter Æ (pronounced "ash"). It is a zero energy wave. Equations will be given in Section 2.

We know cousins of these waves from quantum mechanics (QM). For example, if we picture a one dimensional plane wave coming from the right, and moving to the left until it hits a barrier of infinite potential, what would happen? It would bounce off and double back on itself. It then appears to be a standing wave bobbing up and down. We can still think of it as a single wave traveling in countervailing directions. The Elementary Wave in Fig. 1 is similar in that it consists of one wave traveling in two opposite directions. With the Æ the Schrődinger wave packet is usually absent, so we normally would see only a one dimensional plane wave moving to the left. We are omitting the infinite barrier from this example.

Under specialized circumstances, usually involving collision with a particle, a Schrődinger wave packet abruptly emerges from inside the Æ and carries the particle to the right. In this model there is no wave particle duality. There is however one wave moving in two opposite directions simultaneously. It is a zero energy wave.

Although it is often said that waves always carry energy, that is naive. Schrődinger waves carry no energy. They don't push or pull particles, nor can they do any work. Schrődinger waves carry amplitudes, which are the square root of the probability of a particle being at that location (the Bohm rule). So if Schrődinger waves are zero energy waves, we are simply expanding the boundaries of that concept to include a plane wave moving in the opposite direction. Like many new ideas in mathematics, our model may sound preposterous.

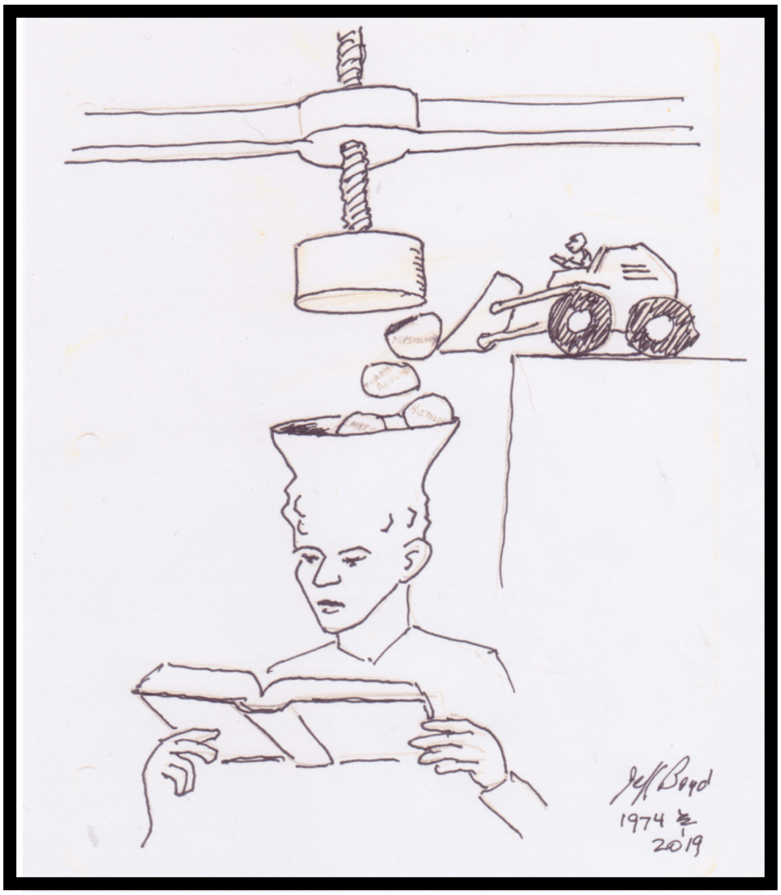

Figure 2: Graduate school is exciting.

The corresponding changes in Quantum Electro-Dynamics will be discussed in Section 7. If you wonder what this article is saying, Fig. 20 in Section 7 is a perfect illustration. If you understand Figs. 19 & 20, then you will understand the heart and soul of this article. This author, by the way, tends to think in pictures, so the graphics in this article are abundant. His friends view this author as a cartoonist, not as a serious mathematician.

1.1 Why being counterintuitive is an advantage

This model is counterintuitive. Why would we be interested in something that sounds absurd to QM experts? Because in sections 3 to 7 of this article we intend to demonstrate that this peculiar model has astonishing effects when applied to those experiments that allegedly "prove" quantum weirdness. Our overall goal is to preserve quantum math, which we regard as the most powerful and accurate science that humans have ever had, but vanquish quantum weirdness.

The fact that our model is counterintuitive is a strength rather than a weakness. Why? Insofar as our model is preposterous, that explains why Einstein and some of the greatest geniuses of all time could not figure this out over the past century. Logical thinking was the downfall of the founders of QM.

If you start with reasonable assumptions you end up with a quantum world that is weird. If you start with weird assumptions you end up with a quantum world that is reasonable. We pay our "weirdness tax" up front. QM does not pay a "weirdness tax" and is therefore penalized forever with a misperception that the quantum world is weird.

In the 34 page published article from which this "essay" is borrowed, we demonstrated that this model changed the meaning of five quantum experiments. Where there was previously quantum weirdness, there is no longer weirdness. The five experiments are:

- Double slit experiments;
- The Purcell effect;
- A quantum eraser experiment;
- Bell test experiments; and
- Quantum Electro-Dynamic experiments.

If we succeeded in showing a vast transformation of the quantum landscape, that would not constitute proof that the peculiar model described in this article should be accepted. All it would mean is that we have discovered an interesting mathematical imbalance. On one side of the scales we would have a small counterintuitive change in how we approach quantum math. On the other side of the scales we would have a pervasive simplification of how Nature appears to work.

Occam's razor might then come to mind.

2 Equations of an elementary wave (Æ)

We will divide our one dimensional Elementary Wave in Fig. 1 into the part traveling left, which we will call Ψ_L, and the part traveling right, which we will call Ψ_R. The point x_0 is where we divide left from right.

Our thinking is guided by an asymmetry. While a wave function might flow in both directions (symmetrical), energy and momentum only flow to the right, not the left. Thus we anticipate a tiger (Schrődinger Wave) moving to the right, but an elongated tail moving left with high speed but no energy. In many ways we are more interested in the tail, because if you control the tail, you control the tiger.

We define x_L to be a location to the left of x_0 (Fig. 1) and x_R to be a position to the right. The vertical axis is amplitude. We define the height and slope of the wave functions to be equal at $x = 0$:

$$x_L = x_R \tag{1}$$

$$\frac{\partial \Psi_L}{\partial x_L} = \frac{\partial \Psi_R}{\partial x_R} = 0 \tag{2}$$

Time, frequency and angular frequency are equal on the two sides:

$$f_L = f_R \equiv f \tag{3}$$

$$\omega_L = \omega_R \equiv \omega = 2\pi f \tag{4}$$

The entire graph in Fig. 1 consists of a wiggly line. The speed of the line itself is tricky. We claim the line moves to the left at light speed, while the wave packet (if it exists) moves to the right at v_R (often less than light speed). We will attribute light speed c to the Ψ_L and v_R to Ψ_R, remembering in the back of our mind that they are comprised of the same substrate, and the substrate is the line moving to the left at c.

Therefore the two wavelengths can be different. We will define

$$\lambda_L = \frac{c}{f} \quad \text{and} \quad \lambda_R = \frac{v_R}{f} \tag{5}$$

Note that $\lambda_L > \lambda_R$ for wave packets moving slower than light.

$$\frac{v_R}{c} = \frac{\lambda_R}{\lambda_L} \tag{6}$$

The substrate Ψ_L carries zero energy. The wave packet Ψ_R also carries zero energy, as we said earlier, but it carries amplitudes for momentum. Variables such as E_R, p_R and k_R exist only on the right side of Fig. 1.

In other words $k_L \neq k_R$. We define

$$k_L = 2\pi/\lambda_L \quad \text{but} \quad k_R = p_R/\hbar \ \text{ where } p_R \text{ is momentum.} \tag{7}$$

Note that p_L and E_L are undefined. As we said before, velocity $v_L = c$, the speed of light. On the other hand, $v_R \neq c$ unless the wave packet moves at light speed, which would only happen if the experiment involved a particle of zero mass, such as a photon.

We now define our two wave functions:

$$\Psi_L = e^{-i(k_L x_L - \omega t)} \qquad \textit{and} \qquad \Psi_R = e^{i(k_R x_R - \omega t)} \tag{8}$$

$$Re(\Psi_L) = \cos(k_L x_L - \omega t) \qquad and \qquad Re(\Psi_R) = \cos(k_R x_R - \omega t) \tag{9}$$

$$Re(\Psi_L) = \cos(\frac{2\pi x_L}{\lambda} - \omega t) \qquad and \qquad Re(\Psi_R) = \cos(\frac{p_R x_R}{\hbar} - \omega t) \tag{10}$$

Note that the ingredients with which to build a Schrődinger wave equation only exist to the right of x_0.

2.1 Deriving the Time Independent Schrődinger Equation:

We define

$$E_R = \text{kinetic energy} + \text{potential energy} \tag{11}$$

$$= \frac{1}{2} m v_R^2 + u = \frac{p_R^2}{2m} + u \tag{12}$$

Taking the second derivative $\partial^2/\partial x_R^2$ of the wave function $\Psi_R = e^{i(k_R x_R - \omega t)}$ (Eq. 8 Right), we get:

$$\frac{\partial^2 \Psi_R}{\partial x_R^2} = \frac{\partial^2}{\partial x_R^2}(e^{i(k_R x_R - \omega t)}) = (ik_R)^2 \Psi_R = -k_R^2 \Psi_R = \frac{p_R^2}{\hbar^2}\Psi_R \tag{13}$$

$$\hbar^2 \frac{\partial^2 \Psi_R}{\partial x_R^2} = p_R^2 \Psi_R \tag{14}$$

Multiplying both sides of Eq. 12, $\left[E = (p_R^2/2m) + u\right]$, by Ψ_R, we get:

$$E\Psi_R = \frac{p_R^2 \Psi_R}{2m} + u\Psi_R \tag{15}$$

and inserting Eq. 14, we get the **Time Independent Schrődinger Equation:**

$$E\Psi_R = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi_R}{\partial x_R^2} + u\Psi_R \ \ \textbf{= TISE} \tag{16}$$

2.2 Deriving the Time Dependent Schrődinger Equation:

The time dependent equation can be easily derived by differentiating our wave equation

$$\Psi_R = e^{i(k_R x_R - \omega t)} \qquad \text{by } \partial/\partial t:$$

$$\frac{\partial \Psi_R}{\partial t} = -i\omega\Psi_R \tag{17}$$

$$\text{We define } E_R = \hbar\omega. \text{ Multiplying that by } \Psi_R \text{ we get:} \tag{18}$$

$$E_R\Psi_R = \hbar\omega\Psi_R \tag{19}$$

$$-\frac{i}{\hbar}E_R\Psi_R = -i\omega\Psi_R = \frac{\partial \Psi_R}{\partial t} \tag{20}$$

$$E_R\Psi_R = -\frac{\hbar}{i}\frac{\partial \Psi_R}{\partial t} = i\hbar\frac{\partial \Psi_R}{\partial t} \tag{21}$$

We can substitute that into the Eq. 16:

$$i\hbar\frac{\partial \Psi_R}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi_R}{\partial x_R^2} + u\Psi_R \tag{22}$$

which is the **Time Dependent Schrődinger Equation, TDSE.**

2.2.1 Wave Packet

Until now we have been focusing on waves of a single frequency f and momentum p_R. We now change that to a model that includes a cluster of frequencies Δf and momenta Δp_R. The reason we do so is because Fig. 1 shows a wave packet moving to the right. In order to construct a wave packet we need a cluster of frequencies that we add into a superposition that exhibits constructive interference in a narrow range of distance (Δx_R).

In order to have a cluster of frequencies on the right side of Fig. 1, we need to have the same frequencies on the left. As we said in Eq. 3, $f_L = f_R \equiv f$. However, there is no wave packet on the left because that is an area in which a superposition of wave equations adds up with destructive interference.

In the remainder of this article we will portray Elementary-Schrődinger Waves as having a nascent wave packet but not an explicit Schrődinger wave packet in most cases. The triggering of a Schrődinger Wave Packet to suddenly appear when the elementary wave approaches a particle, is an unusual event that occurs rarely and under special conditions.

In any volume of space there are a finite number of wave packets but an infinite number of elementary waves.

2.3 Elementary Wave traveling to the left

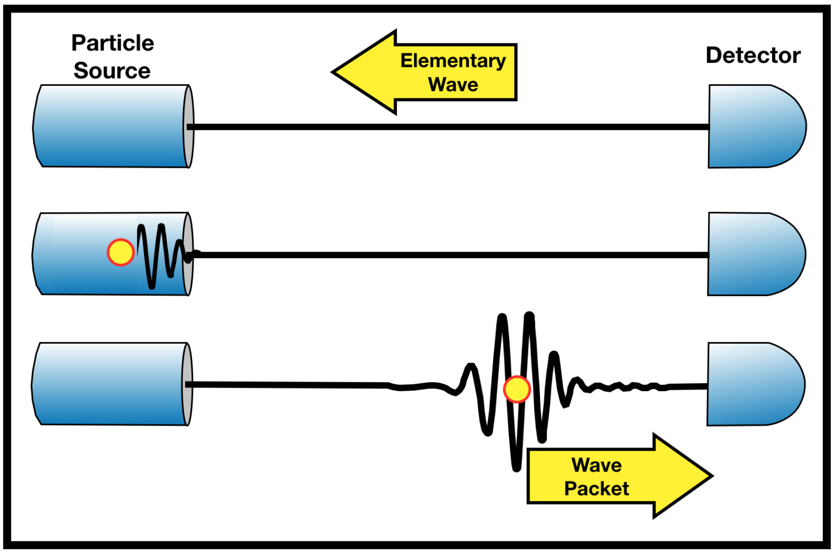

Figure 3: This diagram shows what precedes the emergence of a Schrődinger wave packet. The top row shows a zero energy plane wave moving from the detector leftward to the particle source. That is **the detector's "invitation."** Middle shows a particle about to be emitted from inside the particle gun. Such a particle triggers a Schrődinger wave packet to emerge from inside the incident plane wave. The bottom row shows the wave packet carrying the particle toward the detector. This diagram represents our model of how an Elementary Wave (Æ) works.

In Eq. 8 we stated the wave function for the elementary wave traveling to the left. When that wave equation is combined with the Schrődinger equation of the wave traveling right, you get a compound equation that defines and elementary wave Æ. Compound equations are well known in quantum mathematics. For example in a wave equation for a potential well it is commonplace to have a plane wave defined if $x < 0$ or $|A| < x$, but another wave equation for the well itself (when $0 \leq x \leq |A|$.

According to TEW the world is more inter-connected than QM knows about. **No Schrődinger wave packet can strike a detector unless the detector has "invited" it to do so.** Such an "invitation" is diagramed in the top row of Fig. 3.

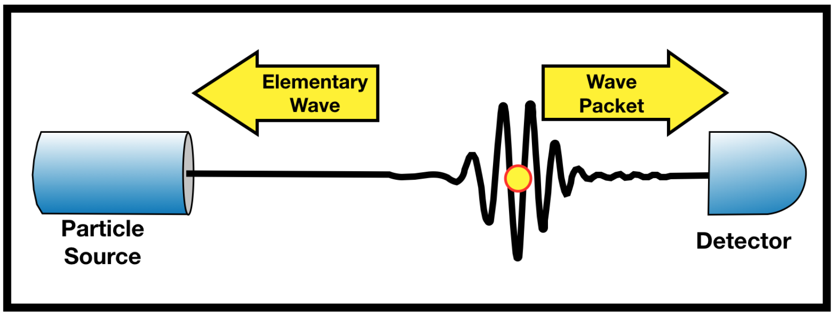

Figure 4: This is a simplification of the previous Figure. This diagram serves as a symbol or condensed version of the previous Figure.

As we said before, the defining feature of an elementary wave is that it carries this intrinsic trigger mechanism for a Schrődinger wave packet to emerge, moving in the opposite direction. Such a trigger is activated when the Æ encounters a particle with precisely the right characteristics, usually involving a particle that has chosen to respond to that specific incident wave.

Such a trigger might be activated if the Æ of frequency f approaches a particle whose De Broglie frequency is $f = E/2\pi\hbar$, and if the particle is about to be launched from a gun, and if the particle makes a random choice of that specific Æ rather than the other competing Æ's. Under those circumstances the wave packet mechanism could be triggered as shown in the middle of Fig. 3. The wave packet would then carry the particle off toward the detector from which that specific Æ is propagating (bottom row of Fig. 3).

In QM Hilbert space is often assumed to be highly abstract, in the stratosphere, in the "space of states." In TEW Hilbert space is interwoven in the Euclidean space of everyday experience. This down-to-earth concept of Hilbert space was developed in one of our earlier publications[Boyd 2019a]

According to TEW, at every point in space there are an infinite number of Elementary Waves traveling in all directions and at all frequencies, at the speed of light. Because they carry no energy, most of them are invisible to our detectors. Our detectors can only see a wave particle Æ-Π (where the symbol "Π" signifies a particle). There is no such thing as a particle without an elementary wave. The intrinsic nature of particles is that they must always be attached to one Æ or another. They can jump from one elementary wave to another. But naked particles, disconnected from all elementary waves do not exist.

Particles can pop in and out of existence, transferring their mass-energy to a new particle, attached to a different Æ.

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